

Bridging in Item Response Theory Models: Improving Latent Trait Estimation in Expert Surveys*

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Latent concepts are ubiquitous in political science. The study of political systems, policies, and belief systems requires careful theorizing and exploration of concepts that are not directly measurable. In particular, constructs such as level of democracy, extent of human rights, and ideology are all latent qualities that we cannot observe directly. Nonetheless, social scientists have theoretical reasons to work with measures of such latent constructs and core questions in our disciplines are virtually impossible to tackle if we restrict the objects of our analyses to concrete observables. Therefore, we often rely on the subjective judgment of experts to provide measures of latent traits.

Of course, there are many ways to leverage expert knowledge for concept measurement. One approach places substantial faith in a small number of experts. For instance, one might use one specialist, or a small panel of judges, who jointly code the set of observations, say politics (Marshall & Gurr 2011). This strategy assumes that a small group of experts have access to the knowledge and information necessary to adequately measure latent traits across the population of interest. Not surprisingly, this assumption may be difficult to meet in cross-national studies where detailed local knowledge is necessary to adequately assess concepts.

*The authors would like to thank Andrew Martin, Kevin Quinn, and the other members of the V-Dem team for helpful suggestions. This research was supported, in part, by National Science Foundation Grant SES-1423944, PI: Daniel Pemstein, and by Riksbankens Jubileumsfond, Grant M13-0559:1, PI: Staffan I. Lindberg, V-Dem Institute, University of Gothenburg, Sweden.

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Thus, in cross-national research, it is common to distribute the load over a large number of experts, and ask each specialist to code a single unit, say a party’s election manifesto (Klingemann, Volkens, Bara, Budge & McDonald 2006). Ideally, this strategy draws from a cross-national pool of experts that can leverage local knowledge to produce detailed and valid measures.

This measurement strategy relaxes the assumption that a small group of experts can evaluate many units, but has the potential to generate differential assessments across units because it implicitly assumes that experts’ understandings of concepts are identical. In other words, these approaches simply assume that each coder is able to produce a valid point-estimate of the latent concept at hand.¹ Multiple raters may have quite different understandings of latent concepts and this variation in conceptualization may be especially exacerbated when experts hail from varying cultural and educational traditions. Indeed, in the context of party manifestos, König, Marbach & Osnabrugge (2013) show that expert ratings exhibit substantial cross-national bias. More generally, people with varying backgrounds are likely to apply different standards when rating concepts (King & Wand 2006). For example, consider the task of rating the extent of media freedom in a country, using a Likert scale running from zero (no freedom) to five (complete freedom). If experts in country A—perhaps because of a long history of freedom of expression—hold media freedom to higher standards than those in country B, then country A expert thresholds will be higher than those of their counterparts. Even within countries, experts’ standards may differ. And, varying standards aside, research shows that people’s responses to Likert scales vary systematically across cultures (Lee, Jones, Mineyama & Zhang 2002). Thus, measurement efforts that rely on cross-national teams of experts need to take the possibility of varying standards seriously.

Both of the aforementioned strategies place substantial faith in each individual expert, and—because they generate only one estimate per observation—provide little leverage over

¹Klingemann et al. (2006) are a somewhat inappropriate example here, because they attempt to train their raters to maximize inter-coder reliability.

potential measurement uncertainty. In essence, therefore, they assume that the expert ratings that they rely upon are error-free, or at least highly reliable.² Expert surveys, where more than one expert provides a rating of each observation, allow researchers to relax this reliability assumption and, at least potentially, produce estimates of measurement error (Bakker, de Vries, Edwards, Hooghe, Jolly, Marks, Polk, Rovny, Steenbergen & Vachudova forthcoming, Fish & Kroenig 2009, Kitschelt 2013). With these approaches, however, aggregating coders' opinions becomes an issue. In most cases these studies focus on measures of central tendency, and ignore precision, or, at best, provide simple summaries of variation in expert codes. Moreover, this approach does little to alleviate concerns about cross-expert equivalency; there is no way of knowing whether experts have the same scale in mind when they provide their codings. For example, we do not know if a Hungarian specialist defines extreme-right political parties in the same way that a Swedish expert might. While expert surveys typically use carefully worded questions that are designed to minimize conceptual slipping, there remains a substantial risk that experts in different contexts might interpret coding rules in systematically different ways.

A more sophisticated approach to aggregating expert ratings explicitly models the measurement process, typically using item response theory (IRT) methods developed by scholars of educational testing (see e.g. Clinton & Lapinski 2006, Clinton & Lewis 2007, Treier & Jackman 2008, Pemstein, Meserve & Melton 2010, Linzer & Staton 2012, Konig, Marbach & Osnabrugge 2013, Schnakenberg & Fariss 2014, Fariss 2014). These models relax two key assumptions implicitly made by traditional approaches to aggregating ratings in expert surveys. First, they allow for variation in rater precision. Second, they allow raters to adopt differing standards of conceptualization. In particular, when raters provide ordinal (Likert scale) ratings of a particular concept, these methods allow for the possibility that different experts have varying thresholds for placing observations into particular categories within the scale. Finally, because IRT methods are grounded in an explicit model of the rating pro-

²Although many of these projects do attempt to verify the inter-coder reliability of their experts or coding teams.

cess, they both aggregate information provided by multiple experts and produce reasoned estimates of uncertainty around aggregate point estimates.

However, these tools are not immune to the problems that plague other strategies for measuring latent social science concepts cross-nationally. In particular, while these methods can, in principle, deal with variation in how experts perceive latent traits, common patterns of data collection in comparative politics—in particular, using disjoint sets of coders across countries—make it difficult for researchers to use IRT models to produce cross-nationally comparable measures of latent concepts. Thus, a persistent problem in the use of these methods has been the issue of calibration/comparability, or lack of *bridging*. Since in many cases the same latent concept is being measured for different units and/or in different times, it is vital to identify *bridges* who cross disjoint sets of coders and provide information necessary to calibrate coder thresholds across units. In our context, these are experts who are capable of rating more than one country. This problem is analogous to that faced by researchers who use IRT methods to estimate ideology from roll call votes and who wish to create *common space* measures across institutions (McCarty & Poole 1995, Bailey 2007, Shor & McCarty 2011) or within institutions across time (Poole & Rosenthal 1997, Groseclose, Levitt & Snyder 1999, Martin & Quinn 2002). Thus, the bridging problem is important both to comparative social scientists that rely on expert surveys and to political scientists who are interested in measuring politicians' and voters' preferences on consistent scales across time and institutional space. Yet, we have little understanding of exactly how much bridging is necessary to achieve scale equivalency across units, and recent work has called into question the extent to which existing common spaces actually achieve scale identification (see e.g. Ho & Quinn 2011).

In this paper we provide a systematic analysis of the issue of *bridging*, motivated by the *Varieties of Democracy* (V-dem) project. This project is, at its core, an expert-survey, in which country experts provide ordinal ratings about dozens of democratic indicators (e.g. barriers to parties, freedom of academic speech, civil society participation), for the 1900-

2012 period. The unit of analysis for each indicator is country-year, for which at least five coders provide ratings. A major selling point of the project is its reliance on a mix of local experts and foreign specialists; generally about half of the raters reside in the country that they rate. But this focus on specialized knowledge comes at a cost; initial survey waves almost exclusively asked experts to provide ratings for only a single country and the length of the survey made both the creation and use of anchoring vignettes (King & Wand 2006) impractical. Subsequent waves have focused on ‘bridge-coding,’ but the search for bridge coders faces a number of practical limitations. First, the specificity of the survey³ means that few potential raters have the ability to provide cross-national coding, especially for the whole time period under consideration. Resources are also an important constraining factor because recruiting bridge-coders and administering surveys is expensive. Thus the V-Dem project faces a problem that is—or should be—generally applicable to a large class of cross-national expert surveys: it needs to recruit bridge coders as efficiently as possible and to evaluate when bridge coding is sufficient to provide cross-nationally comparable estimates of latent traits.

Thus, we seek to answer a set of key questions. First, if we adopt weak prior assumptions about the structure of rater thresholds,⁴ how much bridging is necessary to achieve cross-national comparability in a survey structured like V-Dem? Perhaps more importantly, how do we know when we have enough bridges? Or, failing that, can we at least develop tools to systematically identify cases for which scale comparability is likely to be a problem? Finally, how can we most efficiently select bridges in order to obtain scale comparability across countries? Or, in other words, what set of bridges are likely to provide such comparability at the lowest cost to the project?

³Technically, surveys. V-Dem experts are partitioned into over ten specializations and each set of experts completes a survey specific to her area of expertise.

⁴Weak priors allow raters to have widely varying thresholds. Very strong priors would assume, as most projects involving cross-national expert surveys essentially do, that thresholds are identical across raters. Middle-ground assumptions might maintain that same-country experts have identical, or very similar, thresholds, or that raters with similar backgrounds—education, profession, and so forth—have similar thresholds to one another.

In the next section we describe the IRT model at the heart of this project. Next, we illustrate the bridging problem in V-Dem and then use simulations to explore how much bridging one needs to achieve scale identification when faced with two types of problems that seem common in V-Dem, and probably plague many cross-national expert surveys of social science concepts. Finally, we develop and evaluate a pair of search algorithms designed to select bridges that are most likely to provide information, and thus allow one to correct scale incompatibility problems.

1 An O-IRT Model for V-Dem Data

We need to introduce some notation to keep track of things because the complexity of the V-Dem dataset, which contains ratings of a vast number of indicators across space and time. Moreover, each indicator is rated by more than one judge. This means that there are

- $i \in I$ indicator variables,
- $r \in R$ raters,
- $c \in C$ countries,
- and $t \in T = \{1, \dots, \bar{t}\}$ time periods.

I is the set of indicator variables while i represents one element from that set, and so on. Each of the $|R|$ raters provides ratings of one or more of each of the $|I|$ indicators in some subset of the available $n = |C| \times |T|$ country-years covered by the dataset. Each country enters the dataset at time \underline{t}_c and exits at time $\bar{t}_c + 1$. We'll call rater r 's set of observed ratings/judgments J_r . Each element of each of these judgment sets is an i, c, t triple. Similarly, the set of raters that rated country-year c, t is R_{ct} . In this paper we focus on models for a single indicator, and therefore drop the i indices from our notation. We also restrict our attention to the case where indicator i is ordinal.

For a given indicator we observe a sparse⁵ $|C| \times |T| \times |R|$ array, \mathbf{y} , of ordinal ratings. While our observations are ordinal, we assume that raters make judgments about latent continuous concepts. In particular, we assume that each rater first perceives latent values with error, such that

$$\tilde{y}_{ctr} = z_{ct} + e_{ctr} \quad (1)$$

where z_{ct} is the “true” latent value of the given indicator in country c at time t , \tilde{y}_{ctr} is rater r ’s perception of z_{ct} , and e_{ctr} is the error in rater r ’s perception for the country-year observation. For the cases that she judges (J_r), rater r places a country-year in category k if $\tau_{r,k-1} < \tilde{y}_{ctr} \leq \tau_{r,k}$, where each τ represents a rater cutoff point on the underlying latent scale. The vector $\boldsymbol{\tau}_r = (\tau_{r,1}, \dots, \tau_{r,K-1})$ is the vector of unobserved ranking cutoffs for judge r on the latent scale. We fix each $\tau_{r,0} = -\infty$ and $\tau_{r,K} = \infty$, where K is the number of ordinal categories raters use to judge the indicator. For the moment, say the cumulative distribution function for the rating errors is

$$e_{ctr} \sim F(e_{ctr}/\sigma_r). \quad (2)$$

Combining this assumption, and the assumptions about rater cutoffs described in the above paragraph, with equation 1 implies the following data generating process (DGP):

$$\begin{aligned} \Pr(y_{ctr} = k) &= \Pr(\tilde{y}_{ctr} > \tau_{r,k-1} \wedge \tilde{y}_{ctr} \leq \tau_{r,k}) \\ &= \Pr(e_{ctr} > \tau_{r,k-1} - z_{ct} \wedge e_{ctr} \leq \tau_{r,k} - z_{ct}) \\ &= F\left(\frac{\tau_{r,k} - z_{ct}}{\sigma_r}\right) - F\left(\frac{\tau_{r,k-1} - z_{ct}}{\sigma_r}\right) \\ &= F(\gamma_{r,k} - z_{ct}\beta_r) - F(\gamma_{r,k-1} - z_{ct}\beta_r). \end{aligned} \quad (3)$$

The last two lines of equation 3 reflect two common parameterizations of this model. The first parameterization is typically called multi-rater ordinal probit (MROP) (Johnson &

⁵The majority of raters provide ratings for only one country.

Albert 1999, Pemstein, Meserve & Melton 2010),⁶ while the latter is an ordinal item response theory (O-IRT) setup (Clinton & Lewis 2007, Treier & Jackman 2008). Note, in particular, that $\beta_r = \frac{1}{\sigma_r}$ and $\gamma_{r,k} = \frac{\tau_{r,k}}{\sigma_r}$.⁷ The parameter σ_r is a measure of rater r 's reliability when judging the indicator; specifically it represents the size of r 's typical errors. Raters with small σ_r parameters are better, on average, at judging indicator i than are raters with large σ_r parameters. In the IRT literature, β_r is known as the discrimination parameter,⁸ while each γ is a difficulty parameter. The discrimination parameter is a measure of precision; a rater characterized by an item discrimination parameter close to zero will be largely unresponsive to true indicator values when making judgments while a rater with a discrimination parameter far from zero will be very “discriminating.” The γ and τ parameters are thresholds that control how raters map their perceptions on the latent interval scale into ordinal classifications. The term “difficulty parameter” stems from applications in educational testing where the latent variable is ability and observed ratings are binary (in)correct answers to test questions. The scale identification problem faced by cross-national surveys such as V-Dem is driven by the fact that these γ and τ parameters may—and perhaps are even likely to—vary across raters hailing from different cultural and educational backgrounds.

Completing the model specification requires adopting prior distributions for the model parameters. We will focus on the O-IRT model here. Generally, one sets $z_{ct} \sim \mathcal{N}(0, 1)$ a priori. This assumption (arbitrarily) sets the scale of the estimated latent traits. Second, one assumes $\beta_r \sim \mathcal{TN}(\mu_{\beta_r}, \sigma_{\beta_r}, 0, \infty)$.⁹ Finally, while different authors adopt varying priors for $\gamma_{r,k}$, it is possible to adopt completely uninformative uniform priors for these parameters, subject to the constraint that they remain ordered within raters. When one lacks any

⁶If we assume $F(\cdot)$ is standard normal.

⁷This equivalency breaks down if we allow for β_r parameters less than one. Thus, the O-IRT model is potentially more general than MROP.

⁸Equation 3 makes clear that β_r measures rater precision.

⁹1 is a common choice for μ_{β_r} . One typically chooses a large value for σ_{β_r} , thereby adopting a vague prior. The assumption of truncation at zero equates to assuming that raters agree on which direction is up. While reasonable in our context, this assumption is inappropriate when one uses IRT models to estimate ideology from voting behavior. Relaxing this assumption can complicate the bridging problem significantly because it complicates model identification (Ho & Quinn 2011).

bridging between sets of raters, such a prior will fail to identify the model. Or, in other words, under such a prior, the model lacks sufficient information to set the relative placement of disjoint (with respect to cases rated) sets of rater thresholds, and, in turn, latent trait estimates. Thus, we adopt vaguely informative priors for the $\gamma_{r,k}$ parameters throughout.

At its heart, this model does three things. First, it takes ordinal observations and maps raters' thresholds onto a single interval-valued latent variable. In other words, it provides a reasoned way to deal with a relatively large class of differences in how individual respondents interpret Likert scales. Second, it allows raters to vary in how reliably they make judgments, but largely assumes away the potential for systematic rater biases that are not covered by varying thresholds.¹⁰ This latter point is easiest to see in the MROP version of the model. Specifically, in a standard MROP, one assumes $F(\cdot)$ is standard normal, such that $e_{ctr} = \mathcal{N}(0, \sigma_r^2)$. So, in other words, raters get things right on average, but they make stochastic mistakes where the typical magnitude of mistakes that rater r makes on indicator i is σ_r^2 . So, if $\sigma_r^2 < \sigma_{r'}^2$, then rater r provides more reliable judgments about \mathbf{z} than r' because she makes smaller mistakes on average. Finally, taking differences in rater thresholds and precisions into account, the model produces interval-valued estimates of latent traits—each z_{ct} —accompanied by estimates of measurement error that reflect both the level disagreement between coders on the case in question, and the estimated precision of the coders who rated the case.

2 The Bridge Problem

In practice, we face two related problems when attempting to use O-IRT models to generate cross-nationally comparable estimates of latent traits using the V-Dem data. The general problem, of course, is that, if we choose not to simply assume that raters share similar

¹⁰For instance, the model cannot account for a rater that applies one set of thresholds to one country and a different set to another. Nor does this model capture the possibility that rater precisions might vary over space and time, although the model might be expanded to handle such issues.

thresholds,¹¹ we cannot establish the cross-national comparability of our estimates. And, because the scale of the model is established by the vague prior on the z_{ct} parameters, we know that, in the absence of any bridging, the scale location of scores within countries will be set relative to the empirical pattern of codes provided by raters. If country traits are highly variable and generated randomly from the same distribution, then scales would tend to converge even without bridging, but real traits do not look remotely like a random, highly variable, process. Furthermore, whether or not bridging observations establish scale convergence relies not just on whether or not we have many bridges, but on how bridging observations correspond to empirical patterns in the data. In other words, some bridges are more informative than others and certain countries are much harder to align with a common scale than others. Indeed, our secondary concern is that empirical patterns in regime traits often exacerbate the fundamental scale equivalence problem.

While it is, in general, difficult, to determine whether two cases are sufficiently bridged to obtain scale equivalence, sometimes the lack of sufficient bridging is painfully obvious. When we naively apply O-IRT models to weakly bridged V-dem data, we consistently produce estimates of latent values for Western-European countries that are lower than for countries in which experts codings' on the original Likert scales point to serious democratic deficiencies. This finding is largely a result of two trends in our data. First, coders of Western, established democracies exhibit low variation in their coding. An example of this issue is presented in Figure 1. The top two panels present the yearly average ratings for Denmark on the original ordinal scale for V-Dem's barriers to parties indicator. Country experts clearly agree that there are no barriers to parties in Denmark (higher scores reflect lower barriers), except during a brief period in the 1940s. Contrast this result with the highly variable yearly average of this variable for Mali, shown in the top right panel of the figure. The two bottom panels present the result of fitting an O-IRT model, like that described in section 1, to around 150 countries worth of ratings containing few cross-national bridges. While the

¹¹And fitting O-IRT models to single-country series provides substantial evidence that rater thresholds vary, even within countries.

measurement model results for Mali reflect its transition to a “no barriers” mode (as the z_{ct} estimates cross the upper threshold),¹² the measurement model estimates for Denmark never cross the upper threshold. Thus, even though the country experts agree that the situation with regards to parties’ barriers is flawless in Denmark, the lack of variation in ratings is problematic. Clearly, we lack sufficient bridges to place Denmark and Mali on the same scale. Moreover, a reasonable interpretation of the Danish data is that, because every rater agrees that Denmark rates very highly on this trait in every time period, we should place Denmark near the top of the common scale with high confidence. But, once we relax the assumption that thresholds have consistent meanings across raters, country time-series like Denmark provide virtually no information about scores, because we have no way to nail down what Danish thresholds mean relative to one another, or to coders in other countries. Furthermore, while cases that exhibit sufficient variation to produce estimates that span their local scales may look comparable on visual inspection, without sufficient bridging it is impossible to establish the comparability of country time-series, even when they provide substantial information about coder thresholds within countries.

3 A Simulation-Based Examination of Bridging

3.1 Thresholds Drawn from the Same Distribution

To evaluate the effects of bridging on model fit, cross-national comparability and identification, we preform a number of Monte Carlo experiments. Our first set of experiments examines the “Swiss,” or constant-country problem. These experiments, which examine scale equivalence between two simulated “countries,” proceed as follows:

1. We generate “true” values for each z_{ct} , which represent real country-year latent ability scores. In order to proxy the countries we observe in the V-dem dataset, we focus on two country ‘types.’ The first is what we call a ‘constant’ country (used as proxies

¹²Thresholds in these graphs reflect population means for the $\gamma_{r,k}$ parameters.

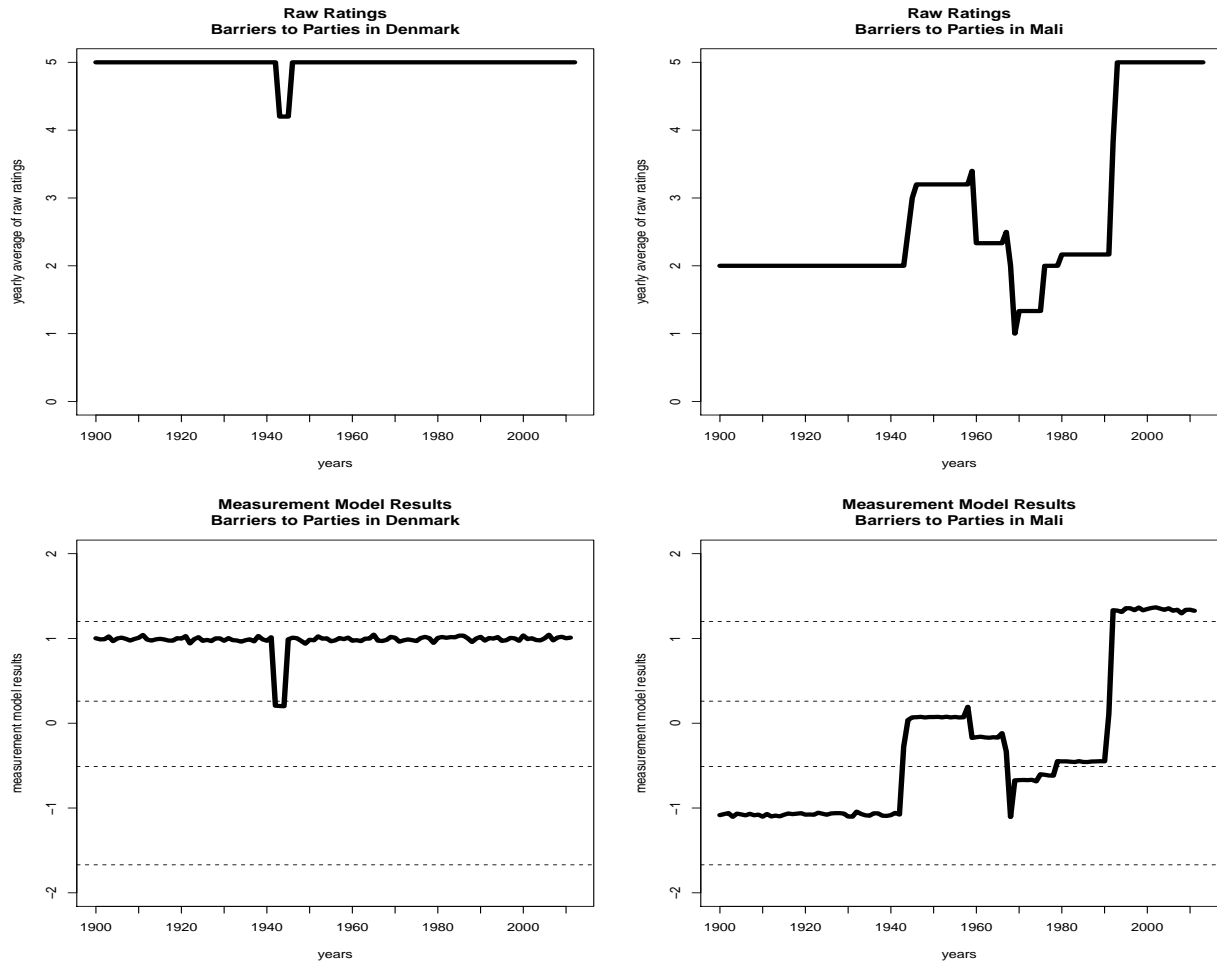


Figure 1: Raw ratings and measurement model results for *barriers to parties* indicator.

for Western-European countries). To simulate these countries, we draw latent ability scores $z_{ct}^{constant} \sim \mathcal{N}(1.8, .05)$. In addition, we generate latent ability scores for countries we dub ‘random.’ These are countries in which coders’ ratings exhibit a high degree of variation, and thus provide us with a substantial information regarding coders’ thresholds. These scores are drawn from a $z_{ct}^{random} \sim \mathcal{N}(0, 1)$ distribution.¹³

2. We also generate $|R| \times K$ coders’ thresholds, $\gamma_{r,k}$. The first threshold, $\gamma_{r,1}$, is drawn from a $\mathcal{N}(-1.5, 0.2)$ distribution. Additional three, $\gamma_{r,k=(2,\dots,4)}$ are drawn sequentially from normal distributions with means $(-5., .5, 1)$ and standard deviation 0.2. For each threshold, we truncate the prior from below by the mean of the $k - 1$ threshold. Note that, while we allow rater thresholds to vary, the fact that we draw them from identical distributions will tend to minimize the role that such variation plays in scale equivalence. Thus, we focus attention on the issue that constant countries pose in these experiments.
3. To complete the latent data, we generate coders’ discrimination parameters such that $\beta_r \sim \mathcal{TN}(1, 3, 0, \infty)$ distribution.
4. Given the above simulated parameters, we use equation 3 to simulate observed ratings data.

For our purposes, bridge-coding is defined as a situation where a coder who ‘originally’ codes one country (for example, an expert of British politics, rating only the UK), also provides ratings for an additional country. As for our stylized, simulated datasets, we generally begin with ‘baseline’ data, in which we simulate data for two countries, over a period of 100 years. This baseline dataset includes five coders who provide ratings *only* for one country, and five who provide ratings for the other. This is a ‘no-bridging’ scenario. In order to

¹³Throughout the paper we use the term *constant country* to denote the country whose latent ability scores were drawn from a $\mathcal{N}(1.8, .05)$ distribution, and ‘random country’ to denote a $\mathcal{N}(0, 1)$ country. Random countries are, of course, an ideal type never actually realized in the V-Dem data. Constant countries, on the other hand, are an ideal type that is lamentably common in nature.

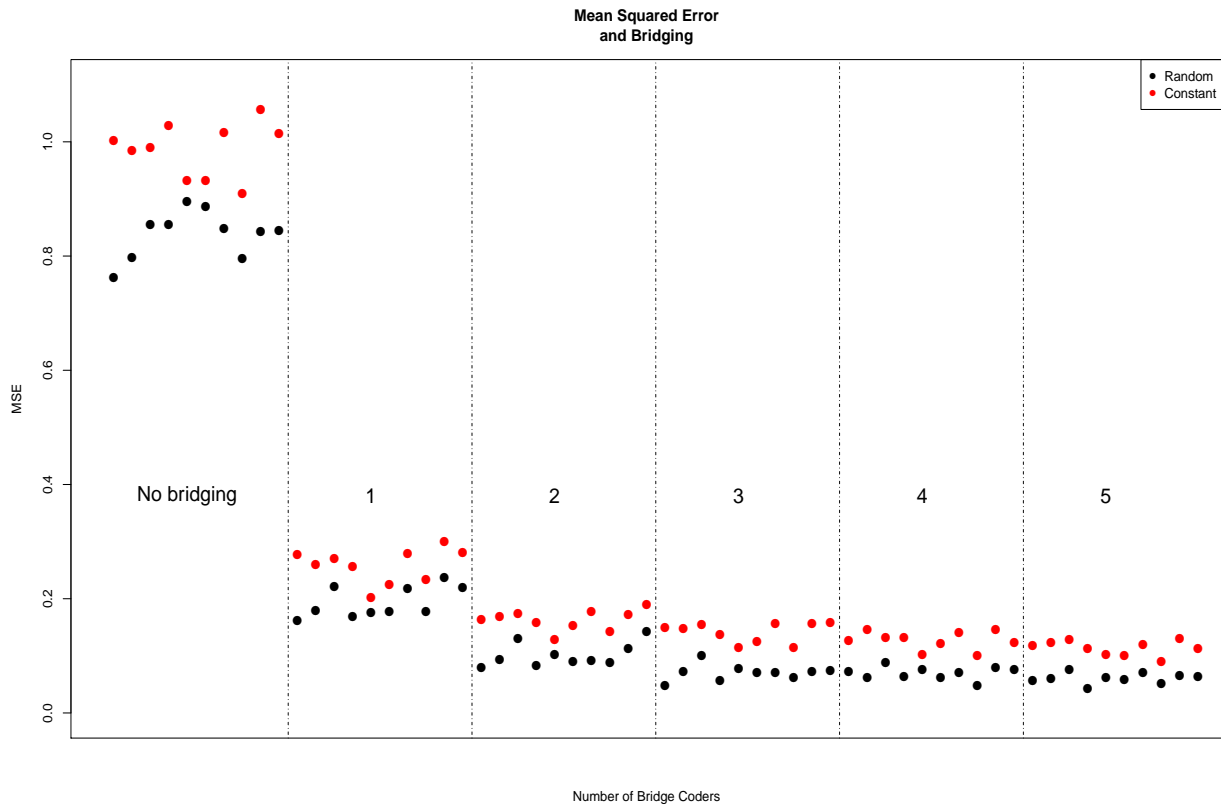
simulate bridge coding, we choose a number of coders from only *one* country, and simulate their ratings for the other. We do this gradually, starting with only one bridge coder, and up until we have ‘full-bridging.’ This is a situation where all five coders from a given country also provide ratings for the other.

We begin our evaluation of the effects of bridging on the mean square error (MSE) between the model’s estimates and the real latent scores. Bridge-coding is simulated for the entire period (100 years). The results of these simulations are shown in Figures 2 and 3. The figure is divided into sub-regions, capturing the different levels of simulated bridge coding (e.g., the left sub-region is a ‘no-bridging’ scenario, and in the fourth, there are three coders from the first country who also rate the second). We replicated each experiment across ten simulated datasets. The figures demonstrate that bridging, done either from the constant to the random country or vice versa, results in a rapid improvement in terms of MSE. Most strikingly, once we simulate one bridge coder, the MSE values for both country types decrease from approximately 1 to 0.3 (in the constant country), and approximately 0.85 to 0.2 (random). Once we add more bridge coders MSE values are further reduced, and reach a plateau of approximately 0.15-2 for constant countries and 0.1-0.15 for the random country with 3-5 bridge coders.

The second criterion we use to evaluate the effects of bridging is the percentage of the latent ability scores z_{ct} which are covered by 95% of the model’s posterior latent scores estimates (coverage). Examination of the Figures 4-5 reveals that regardless of the direction of bridging, even one bridge coder (for the entire period) increases the coverage rates to approximately 90%-100%.¹⁴

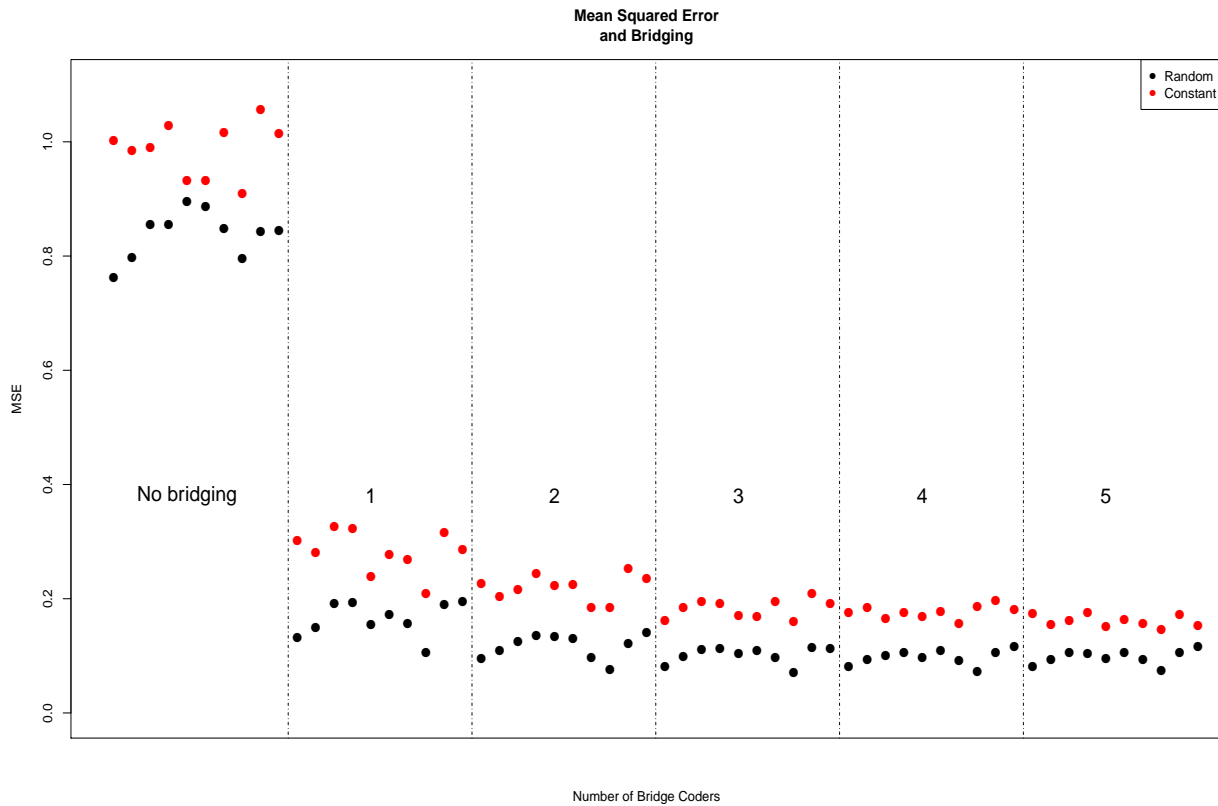
Next we turn to direct comparisons between the model’s parameters and the generated data. We begin by inspecting the z_{ct} parameters. By examining this we hope to assess the degree to which lack of variation in latent scores affects model fit, and the extent to which

¹⁴We repeat those simulations with limited bridging periods (with bridge coding covering only ten years instead of the entire period. The results follow the same pattern, with a smaller magnitude). This implies that we are likely to benefit far more from recruiting a small number of bridge coders to code full time series across constant and varying countries than we are from asking many coders to rate limited cross-sections.



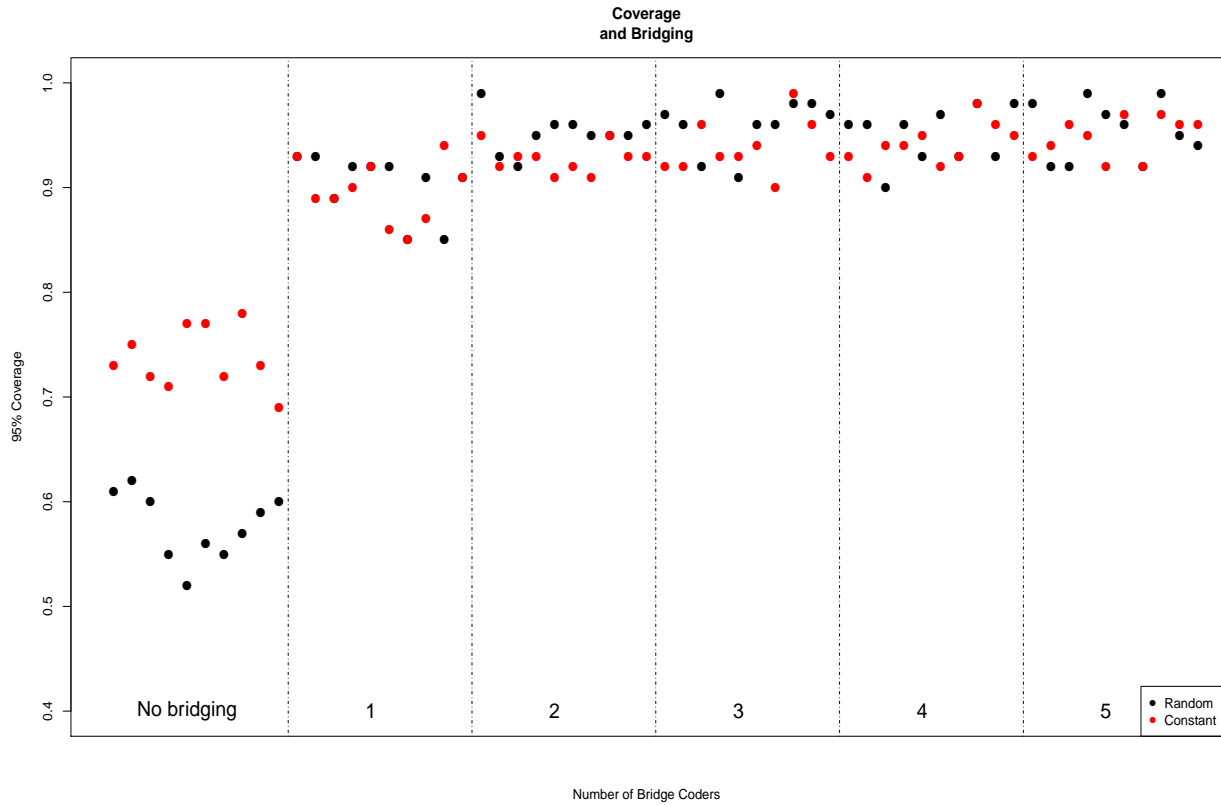
Note: The figure shows the effects of bridging from a constant to a random country. Bridge coding is done for the entire time period (100 years). The vertical lines divide the figure into the bridging patterns, from a no-bridging to full-bridging (i.e. all coders from the constant country also code the random country). In each section of the figure, points represent the MSE for 10 relevant simulations.

Figure 2: Evaluation of bridging from constant to random countries—mean square error of latent scores



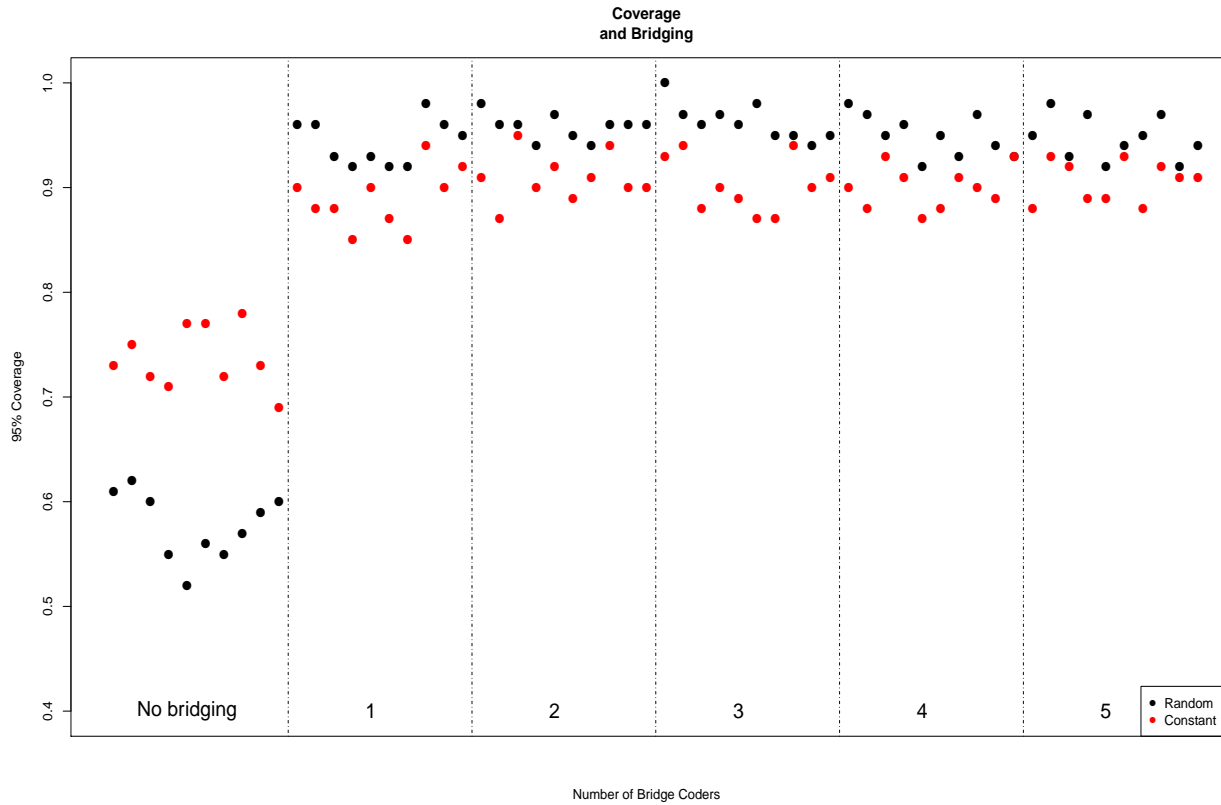
Note: The figure shows the effects of bridging from a random to a constant country. Bridge coding is done for the entire time period (100 years). The vertical lines divide the figure into the bridging patterns, from a no-bridging to full-bridging (i.e. all coders from the constant country also code the random country). In each section of the figure, points represent the MSE for 10 relevant simulations.

Figure 3: Evaluation of bridging from random to constant countries—mean square error of latent scores



Note: The figure shows the effects of bridging from a constant to a random country. Bridge coding is done for the entire time period (100 years). The vertical lines divide the figure into the bridging patterns, from a no-bridging to full-bridging (i.e. all coders from the constant country also code the random country). In each section of the figure, points represent the coverage for 10 relevant simulations.

Figure 4: Evaluation of bridging from constant to random countries–95% coverage



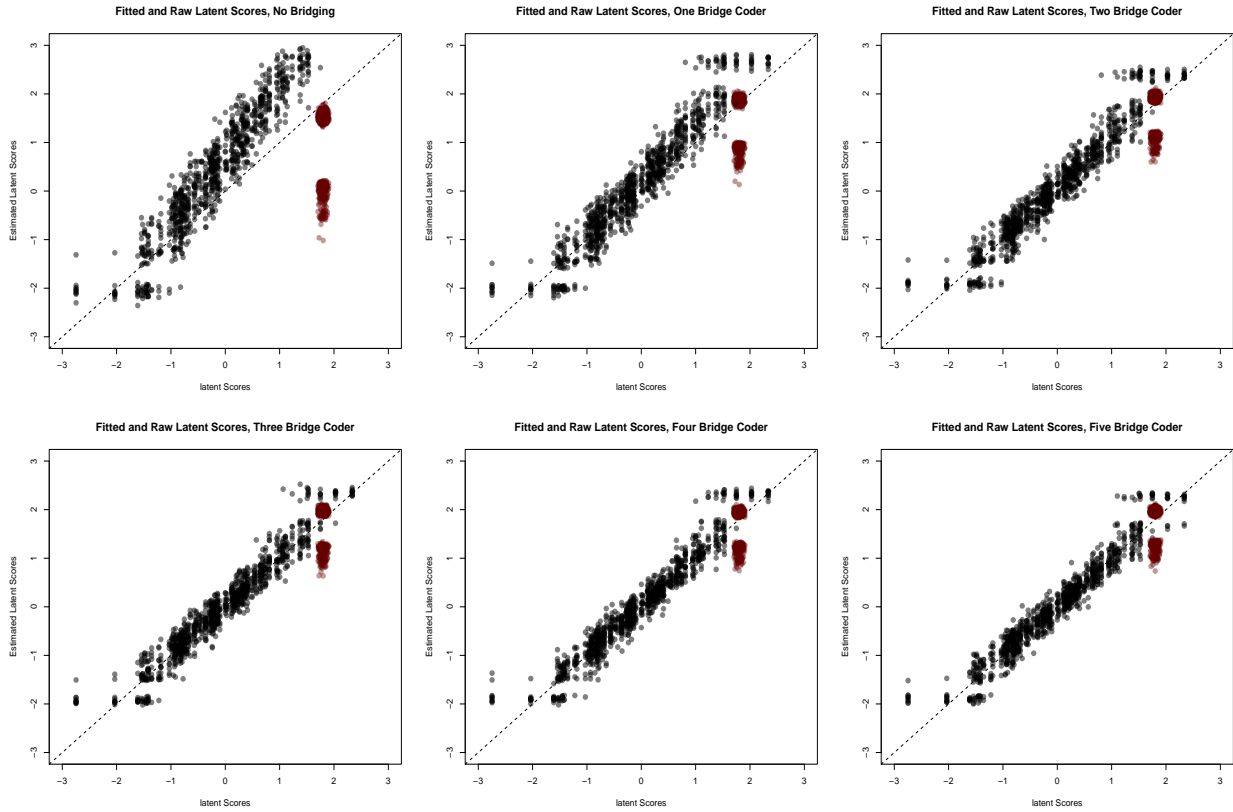
Note: The figure shows the effects of bridging from a constant to a random country. Bridge coding is done for the entire time period (100 years). The vertical lines divide the figure into the bridging patterns, from a no-bridging to full-bridging (i.e. all coders from the constant country also code the random country). In each section of the figure, points represent the coverage for 10 relevant simulations.

Figure 5: Evaluation of bridging from random to constant countries–95% coverage

bridging helps alleviate potential issues. Before performing these direct comparisons, we follow Bafumi et. al (2005) and rescale the model’s fitted values so that they have the same mean and standard deviation as the original latent scores. To present these comparisons systematically, we plot the generated country latent scores against the model’s z parameters in Figures 6 and 7. The first thing noticeable in Figure 6 is that the estimates for the constant countries (in red) cluster in two distinct areas. One cluster is located close to the 45 degree line, and one below it.¹⁵ In addition, the results of the baseline model (upper left panel in both figures) indicate that the model tends to over-predict the latent scores for the random country. This is a result of the relativity of the latent scales and our decision to mean-center estimates to aid in comparability; the rotation of the latent traits is arbitrary. Focusing on Figure 6, we see that the quantity of bridge coding has a strong effect on model fit. Bridging pulls both the over predicted and the under predicted (constant) values toward the 45 degree line. Much like in previous diagnostics, this process is gradual. In the simulation in which all five coders from the constant country also code the random one (lower-right panel), the estimates for the random county are almost entirely clustered on the 45 degree line (apart from a number of outliers at the extremes of the distribution). Under this scenario, the estimates from the constant country are also closer to the 45 degree line.

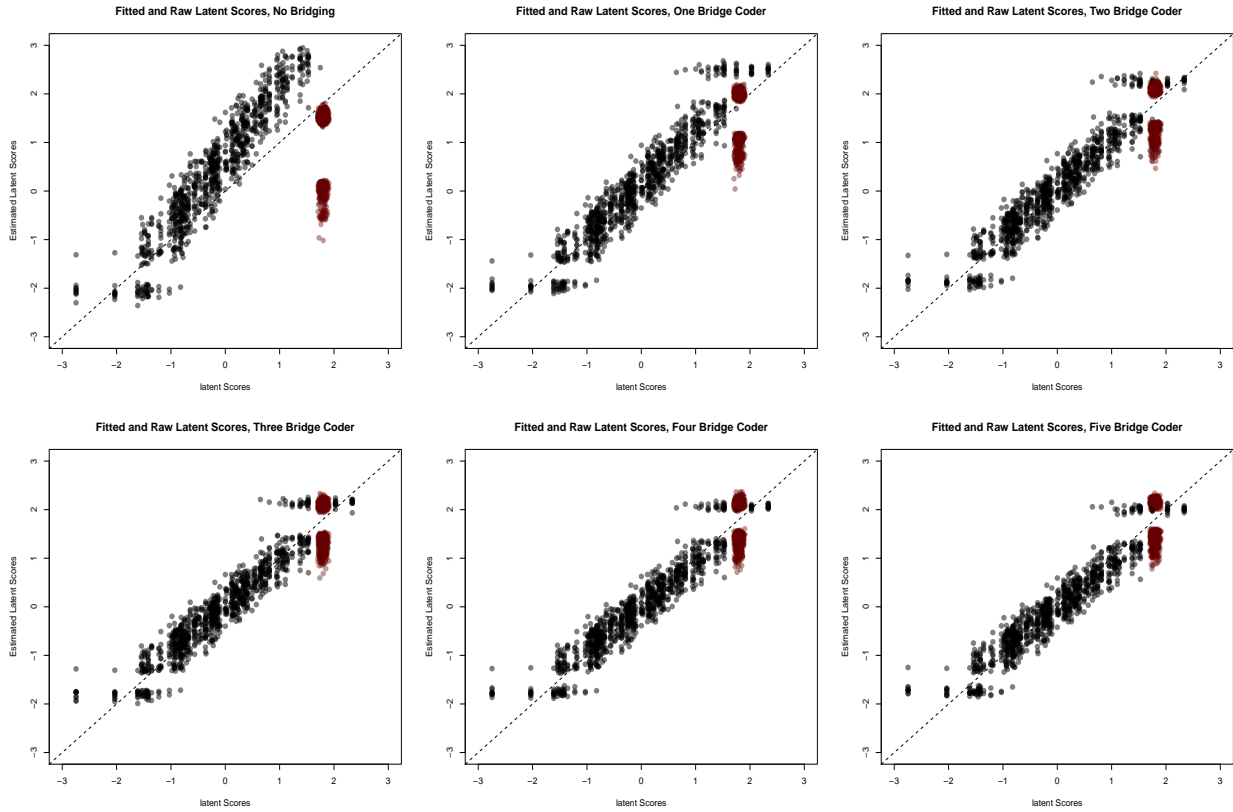
The results presented in Figure 7 match closely with those observed in Figure 6. However, closer inspection of our results reveals that when the bridge coding is done from the constant country to the random one, the fit of the model is improved. As can be seen in Table 1, the MSE for both random and constant countries are lower for constant-random bridging than for random-constant bridging. This result makes a lot of sense because, prior to bridge coding the model knows a lot less about the thresholds of raters in the constant country than it does about those in the random country. In terms of bridge-rater recruitment strategy, we see that we stand to obtain greater benefit from asking “Swiss” coders to code cases with substantial variation than we do from recruiting coders with well-estimated threshold values

¹⁵This looks suspiciously like a multiple-modes problem. A similar pattern does not appear when we use variational approximation to estimate the O-IRT model. Here we use Hamiltonian Monte Carlo.



Note: The figure shows the effects of bridging from a constant (red) to a random (black) country. The upper left panel presents the results for the baseline mode (no bridging). Points depict the results of per simulations per regime (no bridging, one bridge coder, two bridge coders, etc.)

Figure 6: Fitted and real latent score estimates. Bridging is from constant to random countries



Note: The figure shows the effects of bridging from a random (black) to a constant (red) country. The upper left panel presents the results for the baseline mode (no bridging). Points depict the results of per simulations per regime (no bridging, one bridge coder, two bridge coders, etc.)

Figure 7: Fitted and real latent score estimates. Bridging is from random to constant countries

Table 1: Sums of Mean Square Error

	constant country	random country
c-r bridging	1.13 (0.01)	0.60 (0.009)
r-c bridging	1.60 (0.009)	0.97 (0.01)

Note: Sums of mean square errors for model including full bridging pattern (bottom-right panels of Figures 6 and 7) . c-r bridging implies that coders from constant countries also code random the country. r-c stands for the opposite pattern.

to code constant cases.

We now turn to inspecting model fit with regards to the coders' thresholds parameters $\gamma_{j,k}$. As in the previous examples, we compare patterns and quantity of bridging, how these affect the model's predictions, and their relationship with the latent generated data. Figures 8 and 9 summarize these experiments. As always, the no-bridging scenarios yields estimates that are biased. Specifically, the threshold parameters of coders who only rate the random country are over-predicted, while those of coders who only code the constant country are under-predicted. As can be seen in Figure 8, increasing the degree of bridging from the constant to the random country reduces the bias, as more and more of the points fall closer to the 45 degree line. In addition, it is clear that even without a large degree of bridging, the lower thresholds of the constant coders are estimated with minimal bias.¹⁶ Overall, even with full bridging (i.e. when all coders from the constant country also code the random country), there is still bias in the estimates of coders' thresholds. Examination of figure 9 highlights the extent of this issue. The figure shows that when the bridging pattern is from the random to the constant country, bridging has little effect on the bias with which these parameters are estimated. This problem is especially acute when it comes to estimating constant country coders' thresholds .

¹⁶We simulate the constant country such that coders almost give it a perfect, high score. This minimal information is sufficient to estimate the location of the lowest threshold with little bias, but not of other thresholds.

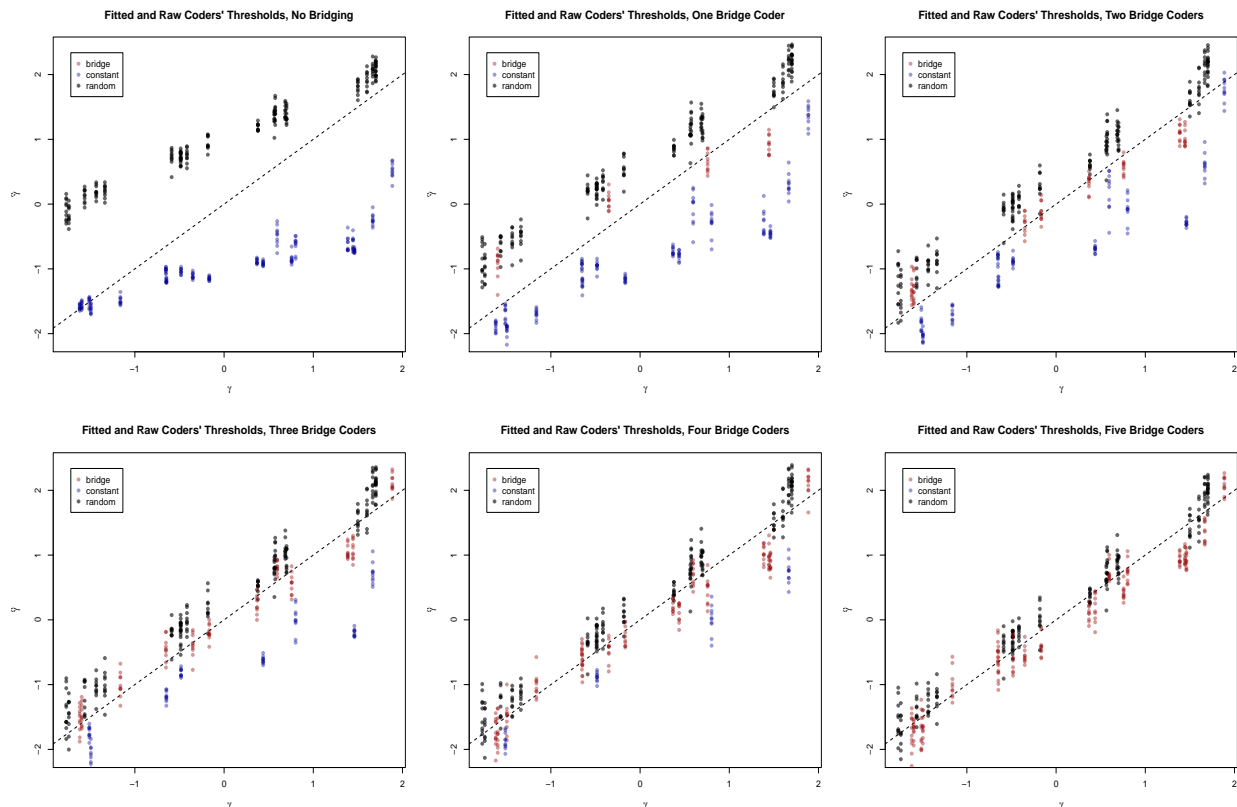


Figure 8: Fitted and real thresholds’ estimates. Bridging is from constant to random countries.

In addition, the lower thresholds of the coders who are originally from the constant country (depicted at the upper panels of Figure 8 in blue dots and in the bottom panels in red, as we simulate more and more bridge coding), are estimated with minimal bias when we simulate full bridging¹⁷

3.2 Thresholds Drawn from Different Distributions

We now turn our attention to situations where latent traits exhibit substantial variation within countries but coders across countries have significantly different thresholds. To simulate the situation where coders from different countries have different thresholds, we generate the coders’ thresholds drawn from different distributions. For the “low threshold” coders, the means of their k thresholds, $\gamma_{\mu,k}^{low}$, are drawn from a $\mathcal{N}(-1, .5)$ distribution. These coders

¹⁷That is, a situation where all countries from country A also code country B.

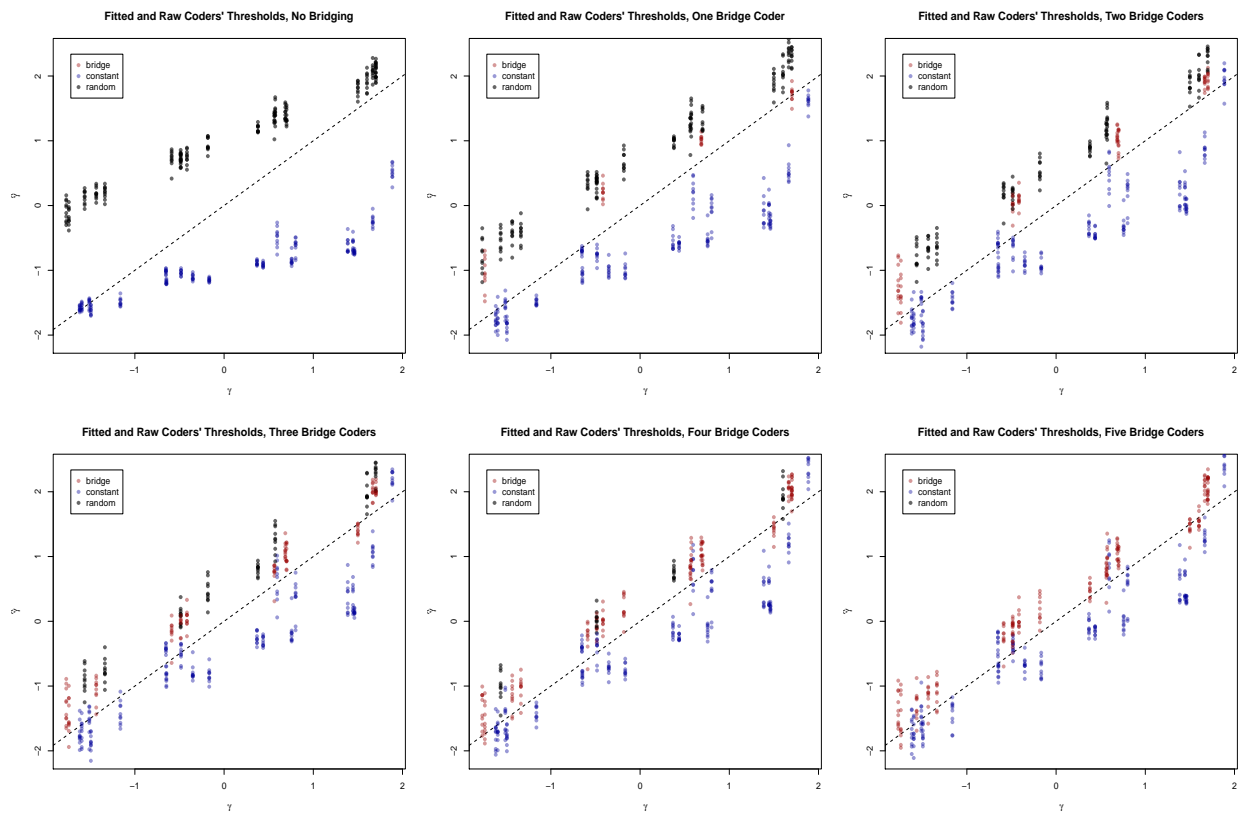


Figure 9: Fitted and real thresholds' estimates. Bridging is from random to constant countries.

are able to discriminate between lower-valued latent scores, but lump high latent values into their top ordinal category. For the “high threshold” coders, the means of their k thresholds, $\gamma_{\mu,k}^{high}$, are drawn from a $\mathcal{N}(1, .5)$ distribution, subject to the ordering constraint. These coders are less able to discriminate low latent traits. This simulates a country where coders consistently tend to be more harsh on their evaluations of the country’s level of political development. These two sets of coders rate two “random” countries, for which the latent scores are both drawn from a $\mathcal{N}(0, 1)$ distribution. The parameters of coders’ discrimination are generated from the same distribution as in the previous simulation.

In the baseline (unbridged) model, five “low threshold” coders rate only one country, while the five “high threshold” coders code the other country. Then we evaluate the effects of bridge-coding by simulating either “low threshold” or “high threshold” coders’ ratings for the other country for the entire period. The MSE of these simulations are shown in Figure 10 and 11. Figure 10 and 11 demonstrate the effects of bridging coding done by “high threshold” and “low threshold” coders, respectively, on the MSE. The Figures suggest that bridging, either done by the “high threshold” or “low threshold” coders, brings a rapid improvement for the “bridged country.” However, the improvement for bridge coders’ original country is not substantial. For example, when a “low threshold” coder does bridging coding for the other country, it provides information about where thresholds of those “high” coders are. In addition, the “high” coders’ are less able to distinguish between latent traits below their lowest threshold, and the bridging coding helps solve this issue. Similarly, the “low” coders cannot distinguish between latent traits above their highest threshold, but since none of the “high” coders does bridging coding for the “low” coders’ country, those high latent traits are still not accurately estimated.

Figures 12 and 13 more clearly illustrate the effects of bridging coding on the country-year estimates. The Figures show that in the baseline experiment, country-years coded by the “low” coders (black dots) tend to be overestimated; while country-years coded by the “high” coders (red dots) are likely to be underestimated. In addition, the high latent traits

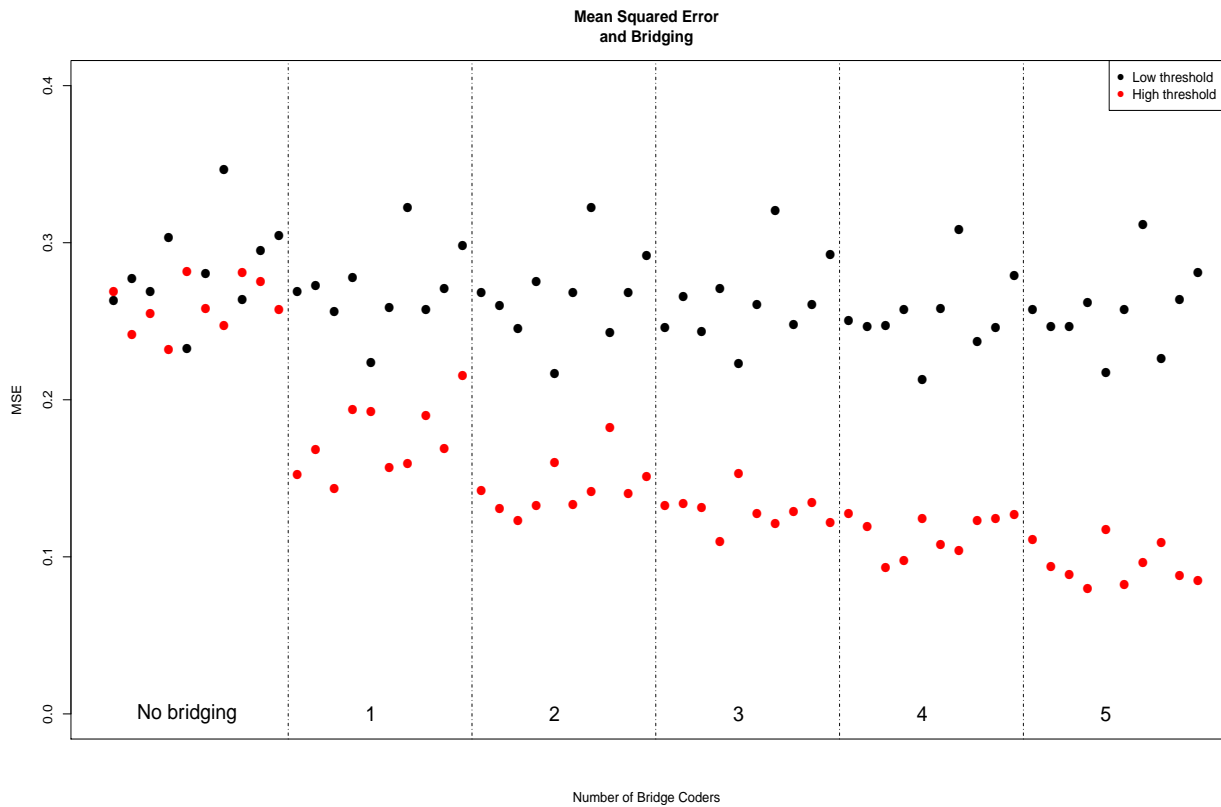


Figure 10: Evaluation of bridging from a country with low coders' thresholds to a country with high coders' thresholds, mean square error of latent scores.

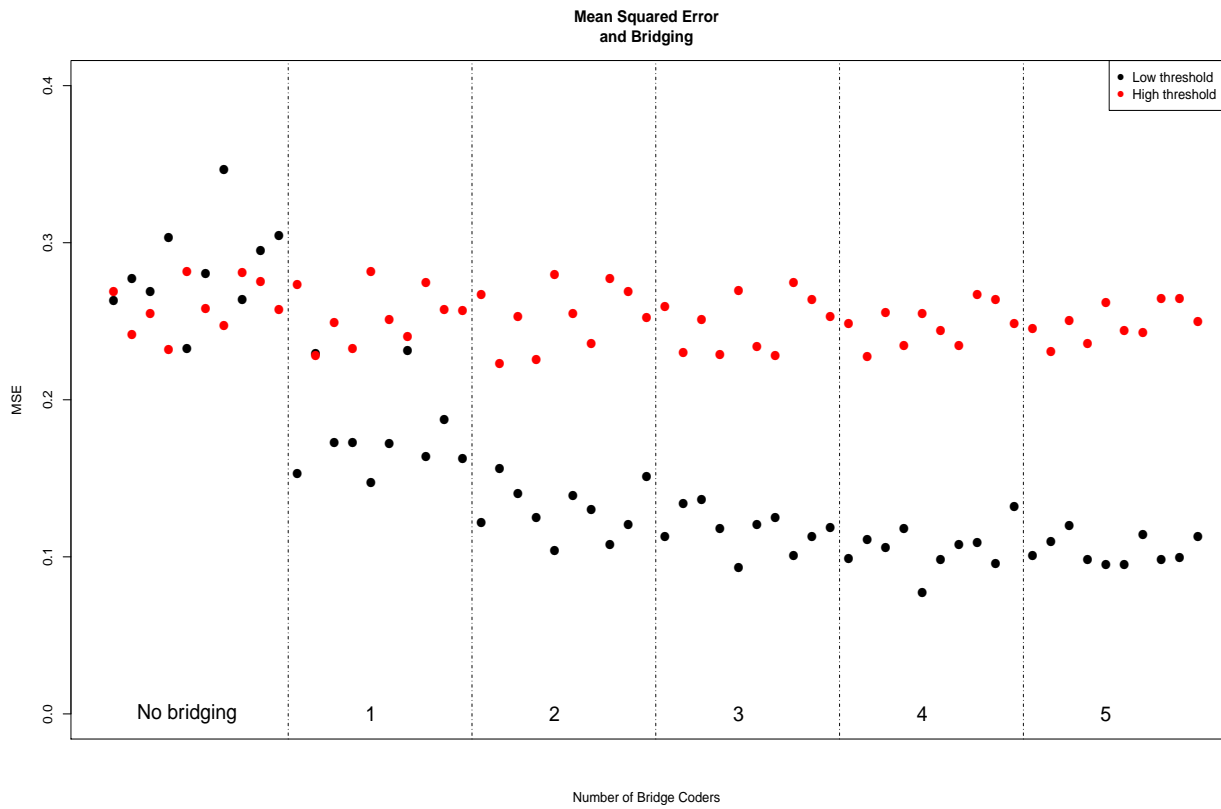


Figure 11: Evaluation of bridging from a country with high coders' thresholds to a country with low coders' thresholds, mean square error of latent scores.

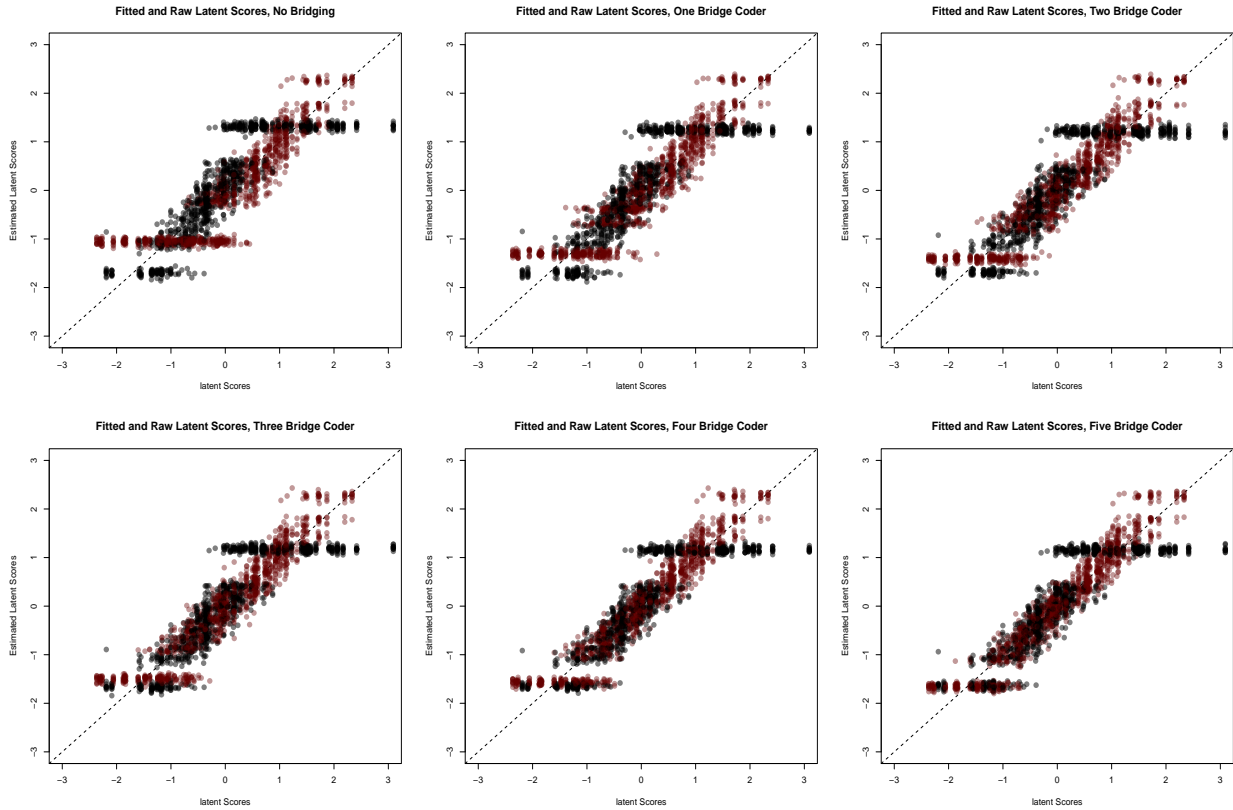


Figure 12: Fitted and real latent score estimates. Bridging is from a random country with low coders’ thresholds to a random country with high coders’ thresholds. Countries coded by low threshold coders are represented by black dots. Countries coded by high threshold coders are represented by red dots.

in the “low” coders’ country are in general poorly estimated, and vice versa. The estimates for both countries move toward the 45-degree line. However, for the bridging countries, the high latent traits in the “low” coders’ country and the low latent traits in the “high” coders’ country are still poorly estimated.

In sum, these experiments imply that, when coders in different countries have significantly different standards—when one set of coders is more strict than the other—we require bridges in both directions to obtain scale equivalence. It also demonstrates that the information provided by limited bridging does not filter through $\gamma_{r,k}$ and z_{ct} estimates as rapidly as one might intuitively expect.

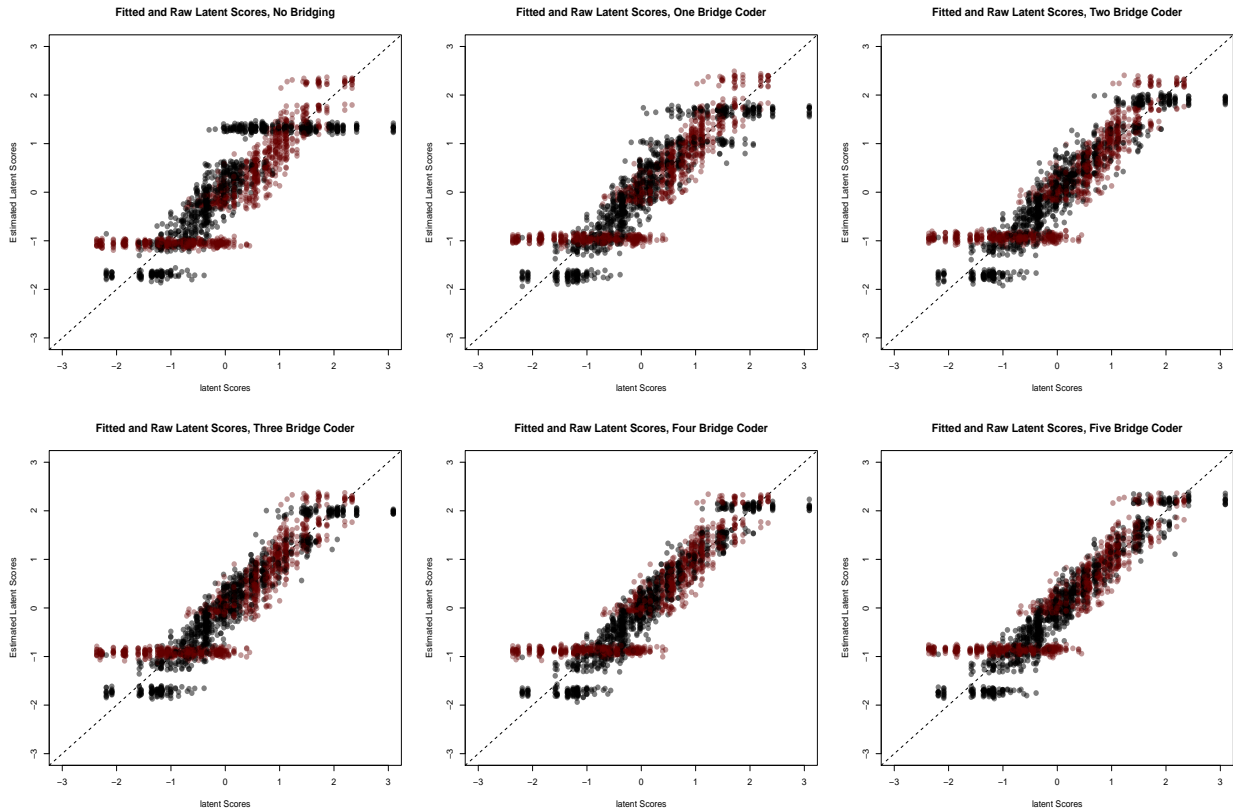


Figure 13: Fitted and real latent score estimates. Bridging is from a random country with high coders' thresholds to a random country with low coders' thresholds. Countries coded by low threshold coders are represented by black dots. Countries coded by high threshold coders are represented by red dots.

4 Efficiently Selecting Bridges

The simulations in section 3 illustrate general patterns in bridge rating efficacy. Ideally, we would ask a significant number of country-coders to provide bridge ratings for long time periods in a second country. But, in projects like V-Dem, where coding demands significant investment from experts, we would like to use patterns in the ratings that we do have to identify potential bridges that are likely to give us the most bang for the buck. Our approach to this problem is motivated by the literature on computerized adaptive testing (CAT).¹⁸

In the testing context, CAT techniques seek to present questions to a student in an order that identifies that student’s latent ability on a particular dimension as quickly as possible. This shortens test time—or survey length—conserving resources. Here, students are analogous to country-years and questions take the place of raters. One generally assumes that question (rater) parameters—each $\gamma_{r,k}$ and β_r —are known and iteratively chooses questions, based on their estimated parameters that minimize the expected posterior variance in the estimate of the student’s latent ability. First, CAT algorithms prioritize highly discriminating questions. Then questions are selected based on the relationship between item thresholds and the current estimate of the student’s latent ability. Intuitively, this generally involves presenting a student with a moderately difficult question to begin with, presenting a harder (easier) question if the student does well (poorly) on the first question, and repeating the process until the estimate of the student’s latent ability reaches some desired level of precision. A common strategy for item selection uses the MEPV (minimum expected posterior variance) criterion. Specifically, for a single student with latent trait, z , at each step in the process, the algorithm chooses the question, q , that meets the condition

$$\operatorname{argmin}_q \left[\sum_{k \in 1}^K p_q(y_q^{i+1} = k | \mathbf{y}^i) \operatorname{Var}(z | \mathbf{y}^i, y_q^{i+1} = k) \right] : q \in \mathbf{Q}_i \quad (4)$$

¹⁸See Montgomery & Cutler (2013) for an introduction to this literature aimed at survey researchers in political science. Choi & Swartz (2009) provide an overview of CAT item selection methods for ordinal items.

where $p_q(y_q^{i+1} = k | \mathbf{y}^i)$ is the posterior predicted probability, prior to answering question q , that the student’s answer to question q achieves a score of k , $\text{Var}(z | \mathbf{y}^i, y_q^{i+1} = k)$ is the posterior variance of the student’s latent ability, conditional on her previous performance and achieving a score of k on question q , and \mathbf{Q}_i is the set of remaining potential questions at the i th step of the process (Choi & Swartz 2009).

4.1 Computerized Adaptive Bridging

Our problem differs from the traditional CAT setting in a number of ways. Most importantly, we cannot assume that our current parameter estimates are valid. Our primary goal here is not to reduce uncertainty, but rather to find and eliminate bias. If latent trait estimates are biased at step i , then a bridging strategy based on MEPV has the potential to erroneously increase our certainty in both biased threshold estimates and biased latent traits. A bridge-selection strategy based on MEPV chooses bridges that are most likely to reinforce existing model estimates. Nonetheless, the logic of CAT suggests a strategy for efficient bias detection and correction. When a bridging problem exists, good bridges will substantially alter parameter estimates, particularly z_{ct} and $\gamma_{r,k}$ values. This is because good bridges are cases in which a bridge rater behaves in a way that is highly inconsistent with the model’s expectations. In a sense, we are looking for bridges that have the potential to *surprise* us, given the current model.

We consider two greedy algorithms¹⁹ that work by iteratively seeking “surprising” bridges. The MAM, or maximum average MSE, algorithm selects bridges at each iteration that have the highest potential to alter estimates, according to the existing model. Formally, at each

¹⁹A greedy algorithm attempts to find a globally optimal solution to a given problem by iteratively making locally optimal choices. In this context, this means choosing individual bridges that maximize (or minimize) some criterion at each step. In other words, these algorithms do not look multiple steps ahead. We focus on greedy algorithms here, both because of their computational tractability, and because they match the process of iterative bridge recruitment that V-Dem must follow in practice.

step in the algorithm, MAM selects the bridge that meets the condition

$$\operatorname{argmax}_{\{c,t,r\}} \left[\frac{1}{K} \sum_{k=1}^K \left(\frac{1}{|C| \times |T|} \sum_{c,t \in C \times T} [E(z_{ct} | \mathbf{y}^i, y_{ctr}^{i+1} = k) - E(z_{ct} | \mathbf{y}^i)]^2 \right) \right] : \{c, t, r\} \in \mathbf{B}_i \quad (5)$$

where \mathbf{B}_i is the set of potential remaining bridges at step i . This approach iterates over potential bridges and chooses the bridge that maximizes the mean square error between current and potential estimates, averaged across potential bridge rater responses.

Another approach to selecting potentially surprising bridges focuses on the posterior predictive distribution. The MALPD—or maximum average local probability difference—algorithm iteratively chooses bridges that have the highest potential for local positive change in posterior predicted probability, on average. In particular, at each step, MALPD selects the bridge that meets the condition

$$\operatorname{argmax}_{\{c,t,r\}} \left(\frac{1}{K} \sum_{k=1}^K [p_{ctr}(y_{ctr}^{i+1} = k | \boldsymbol{\theta}^{i+1}) - p_{ctr}(y_{ctr}^{i+1} = k | \boldsymbol{\theta}^i)] \right) : \{c, t, r\} \in \mathbf{B}_i \quad (6)$$

where $\boldsymbol{\theta}^i$ is the vector of fitted model parameters given \mathbf{y}^i and $p_{ctr}(\cdot)$ is the posterior predictive distribution for case $\{c, t, r\}$. Intuitively, this algorithm looks for bridges that have the potential to elicit ratings that make a lot more sense after the fact than they do before they are observed. This is a potentially fruitful approach because, absent bridging, the model may imply that certain potential ratings are very unlikely, but, after observing such a rating, may judge it quite unremarkable. For example, consider a potential bridge which involves a coder from a high-threshold country rating a case that coders in a low-threshold country have routinely judged to be at the top of the scale. Absent bridging, the model would reasonably predict that the high-threshold coder would also rate that observation highly. But a low rating by the bridge coder should encourage a large shift in coder threshold estimates, and this previously unlikely rating would look quite reasonable in the context of the re-calibrated model.

In principle, we should be able to run either of these algorithms until the observed

MSE between parameters from the pre- and post-bridge model fits falls below a desired threshold. Moreover, we can easily adapt these algorithms for constrained search, by altering the contents of \mathbf{B}_i . For instance, while an expert on parliamentary politics in Ecuador might, in principle, provide a very informative bridge were she to rate legislative corruption in Bangladesh in 1964, we are unlikely to find an Ecuadorian specialist with detailed knowledge of Bangladeshi politics in the middle of the last century. Similarly, it is unlikely to be practical to recruit bridges one at a time in most cross-national expert survey projects. Thus, rather than attempting to recruit only the single bridge that maximizes equations 5 or 6, we could use this approach to identify the set of n most potentially informative bridges, from a set of plausible potential bridges, at each step.

The proposed algorithms for efficient bridge search are very computationally expensive because they require us to generate posterior estimates for the model parameters for each potential outcome of each potential bridging rating at each step of the search. These computational needs quickly become prohibitively expensive when we use MCMC techniques—even ones that generally converge quickly, like Hamiltonian Monte Carlo (HMC)—to estimate model parameters. Standard CAT applications only require the estimation of a single parameter, making numerical integration a practical approach (Montgomery & Cutler 2013); here we need to estimate full multi-parameter models, and solve high-dimensional integrals, making this solution a non-starter. Therefore, we use a variational approximation algorithm to estimate model parameters at each step (Jordan, Ghahramani, Jaakkola & Saul 1999).²⁰ Appendix A provides details.

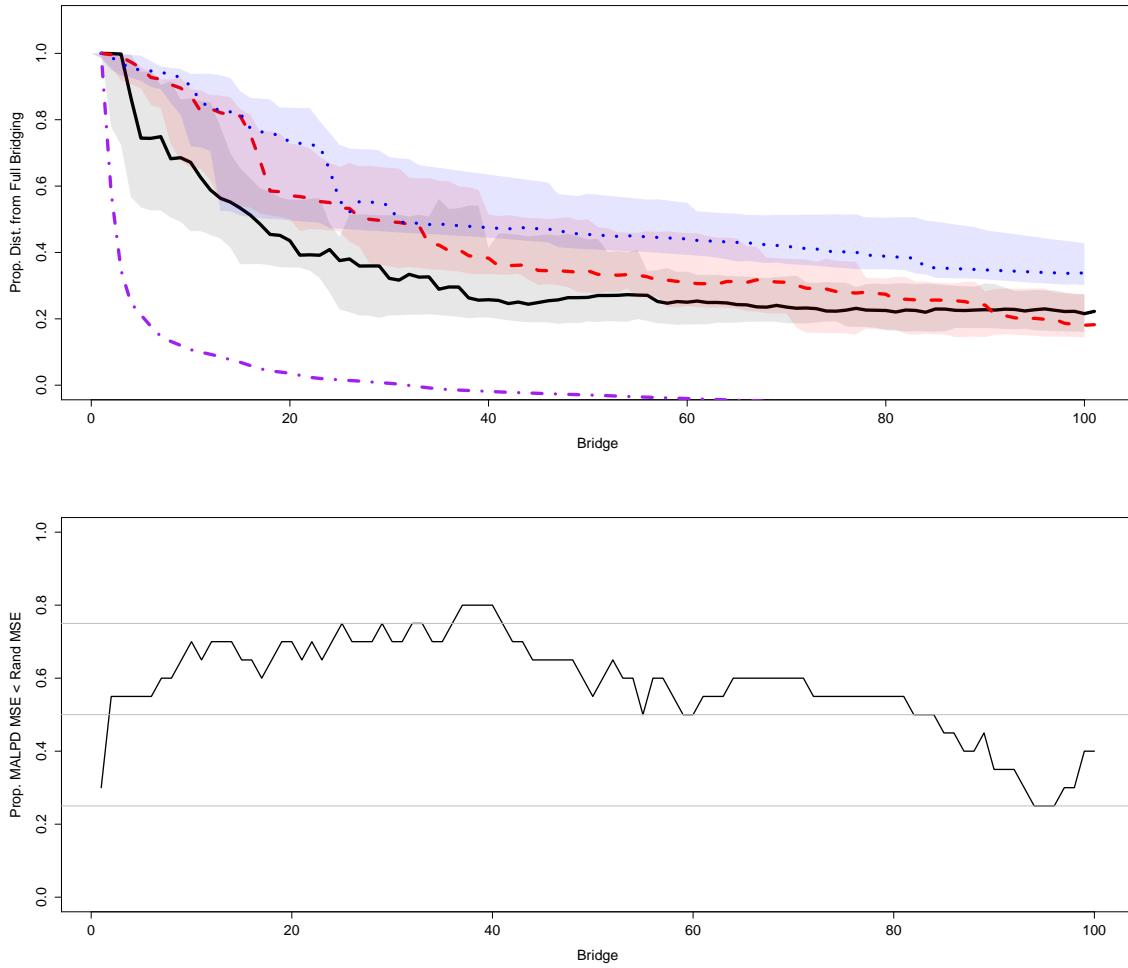


Figure 14: Comparing CAT-Inspired Bridge Selection Algorithm.

4.2 Evaluating the MAM and MALPD Algorithms

Figure 14 depicts the results of a monte carlo experiment that compares the performance of the MAM and MALPD algorithms to random bridge selection and a best-possible greedy algorithm. We simulated 20 two-country datasets using the same procedure we described when examining the constant coder problem in section 3.1. Then we simulated three search process. Each process sequentially added 100 bridges to the unbridged dataset. After each bridge was added, we calculated the mean square error between the “true” distribution of latent traits and the estimates provided by the O-IRT model. In the first search process, we chose bridges at random. In the second, we used the MAM algorithm to sequentially select bridges that had the maximum average potential to alter latent trait estimates. The third process used the MALPD algorithm to iteratively add bridges to the dataset. Finally, best-possible algorithm always chose the bridge that minimized the MSE between true and estimated latent traits, at each bridging step.

The top panel in figure 14 plots the iterative median (across the 20 monte carlo simulations) proportional distances from fully bridged mean square error, with respect to true latent country-year values, for the three bridge selection algorithms.²¹ The bottom panel in figure 14 shows the proportion of monte carlo simulations where the MALPD algorithm produced a lower MSE, at each bridging step, than random selection. Of the three algorithms tested, MALPD shows the most promise. The median MALPD MSE is halved within 17 bridges, something that random selection does not do until the 28th bridge, and which the MAM approach requires 31 steps to achieve. MALPD achieves a median 70 per cent reduction in proportional MSE at 35 bridges, compared with 72 for random bridging. The MAM

²⁰Variational approximation techniques were developed by computer scientists and their descriptions of these methods tend to use jargon that is unfamiliar to social scientists. Ormerod & Wand (2010) provide an overview of these methods, aimed at statisticians, using language that should be familiar to social scientists with statistical training. Grimmer (2010) provides an introduction using examples drawn from political science.

²¹We use this measure to standardize improvement across simulated datasets with varying initial and fully bridged MSEs. So, for simulation s , at bridging step i , we calculate $d_{s,i} = (\text{MSE}_{s,i} - \text{MSE}_{s,\text{full}}) / (\text{MSE}_{s,0} - \text{MSE}_{s,\text{full}})$. Then, for each i , we calculate $\text{median}(\mathbf{d}_s)$.

algorithm does not achieve this target even after adding 100 bridges. The typical MALPD process also outperforms both alternatives until around 90 bridges have been added. By this point, MALPD and random bridging produce essentially indistinguishable results, and both have eliminated about 75 per cent of the MSE difference with full bridging, while MAM trails behind by about 10 percentage points.

Nonetheless, while MALPD outperforms both MAM and random guessing, the improvements it shows are somewhat modest. In particular, it substantially under-performs the best possible greedy algorithm, which obtains a 90 per cent reduction in MSE after roughly 20 bridges. Of course, the best greedy algorithm achieves an ideal that may be out of reach for algorithms that must rely on observables. We are, nonetheless, currently studying the best greedy algorithm in order to determine how to best approximate its behavior using observable information. We are also developing a series of other potential bridging algorithms—both greedy algorithms that maximize other objective functions and more advanced algorithms that use standard search-tree pruning techniques to look down the bridging tree efficiently (see e.g. Russell & Norvig 1995, ch. 4, 5). Finally, we are expanding our simulation studies to consider other common bridging problems, including the low/high threshold problem discussed above.

While developing tools for efficient bridge selection is useful, in and of itself, we also hope to use these tools to develop methods for evaluating the extent to which bridging problems threaten inferences from extant datasets. Logically, the ability to identify critical bridges should provide a way to quantify the potential for bridging failures in fitted IRT models. When potentially critical bridges exist, a fitted model runs a high risk of scale identification problems; when few such bridges are available then we can be confident that no scaling issues exist. Thus, as we improve our algorithms for efficient bridge selection, we are concurrently examining their potential as diagnostic tools.

5 Conclusion

This paper provides perhaps the first systematic analysis of *bridging* in the context of item response models for cross-national expert surveys. We believe this to be an issue that is applicable to a large number of studies, which build upon the ability of country experts to provide detailed information about latent political concepts. While country experts' knowledge is a rich source of data, using this source brings about serious challenges in terms of cross-rater comparability, which, thus far, have not been thoroughly addressed in the literature. Furthermore, while the question has been more extensively examined in the context of measuring ideological common spaces, we lack general tools for effectively assessing bridging quality in latent variable models. Thus, exploring how patterns of bridging affect scale comparability in the sorts of models we examine here has implications for a wide array of work in the discipline.

Of course, our current study is motivated by the data collected as part of the *Varieties of Democracies* project. Therefore, we have focused on patterns of bridging failure that are relevant to V-Dem. We address the issue of bridging low-variance, 'constant' countries, by performing a number of Monte Carlo simulations in which we gradually increase the degree of bridging to, and from, these countries. Our results demonstrate that bridging has a large impact on the model's parameters. First, the results indicate that bridging brings about a rapid decrease in the mean square error of the latent ability scores, and an increase in credible interval coverage. In our simulated data, even one bridge coder (a coder who rates both countries for the entire 100 year period), brings about a reduction of about 70% in MSE, and a 30%-50% increase in coverage rates. However, it is important to note that the pattern of bridging has consequences. The results show that recruiting a limited number of bridge coders who rate a large period in the bridged country is a much better way of reducing bias than obtaining a large number of coders who only rate a small number of years.

In conjunction with examining MSE and coverage, a direct comparison of the model's estimated parameters with the data we generated yields an additional important finding.

Specifically, when it comes to estimating countries' latent ability scores (z_{ct}), bridging from a constant country to a country with substantive variation provides a larger reduction in model bias. When we inspect the effects of bridge coding on estimation of thresholds ($\gamma_{j,k}$ parameters), it becomes evident that a constant-random bridging pattern is a much better way of reducing bias and identifying coders' thresholds than a random-constant bridging pattern.

Next, we tackle the issue of cross-rater thresholds' comparability. We know that in many cases, raters vary greatly in their thresholds. Some coders might be described as lenient, i.e., they have very low thresholds for democratic indicators, while others may be tough, and tend to high higher standards in terms of their rating. We address this issue by drawing thresholds from two distributions with different means. Simulating bridging for the two types of coders shows that bridging brings about an improvement in terms of MSE and of coverage, but that this improvement is limited. Specifically, we gain improvement in estimating z_{ct} parameters from the country from which we simulate the bridging, but not for the target country. Overall, our experiments show how serious the varying threshold problem can be, and that a very large amount of bridging (possibly from both directions) is necessary in order to overcome bias that results from this issue.

Finally, we propose and evaluate an algorithm for efficiently choosing bridge raters and demonstrate some promising results. Overall, the paper clearly shows that bridging (or lack thereof) matters. Ignoring this issues leads to misleading inferences, both in terms of latent ability estimates, and in terms of thresholds (difficulty) parameters. Moreover, achieving cross-rater comparability is a complicated task. Our analysis demonstrates that a large degree of bridging is necessary to bring about an improvement in biases that result from low variation in coders' rating, and from their varying conception of gradation in the latent concept that is being measured.

A Variational Approximation for O-IRT

Variational approximation algorithms allow one to approximately estimate an intractable density function, $p(\boldsymbol{\theta}|\mathbf{y})$, by iteratively approximating the target density in terms of a more tractable density, $q(\boldsymbol{\theta})$, in a way that minimizes the Kullback-Leiber divergence between q and $p(\cdot|\mathbf{y})$. In a common approach, known as a mean field approximation, one selects an approximating density q from a parametric family such that, for some parameter partition $\{\boldsymbol{\theta}_1, \dots, \boldsymbol{\theta}_M\}$, $q(\boldsymbol{\theta})$ factorizes into $\prod_{i=1}^M q_i(\boldsymbol{\theta}_i)$ (Ormerod & Wand 2010). Interestingly, Ormerod & Wand (2010) show that mean field approximation and Gibbs sampling are closely related. In particular, they show that the optimal sub-densities q_1, \dots, q_M for minimizing the Kullback-Leiber divergence between q and p are

$$q_i^*(\boldsymbol{\theta}_i) \propto \exp [E_{-\boldsymbol{\theta}_i} \log p(\boldsymbol{\theta}_i|\boldsymbol{\theta}_{-i}, \mathbf{y})]. \quad (7)$$

In other words, these optimal approximating distributions are functions of the full conditional distributions of the parameters of p . Thus, it is generally straightforward to adapt any Gibbs sampling algorithm, which simulates from p by iteratively drawing from the full conditional distributions of p , into a mean field approximation algorithm. This is helpful in this context because researchers have extensively developed Gibbs sampling algorithms for IRT models (see e.g. Johnson & Albert 1999).

A.1 A Mean Field Approximation Algorithm for the O-IRT Model

We develop a mean approximation algorithm for the O-IRT model described by equation 3, subject to the following priors:

$$z_{ct} \sim \mathcal{N}(0, 1), \beta_r \sim \mathcal{N}(\mu_\beta, \sigma_\beta^2), \text{ and } \gamma_{r,k} \sim U(-2, 2) \text{ s.t } \gamma_{r,1} \leq \dots \leq \gamma_{r,K-1}.$$

In particular, we construct an approximating distribution, q , such that

$$q(\boldsymbol{\theta}) = q_{\tilde{y}_{ctr}}(\tilde{y}_{ctr})q_{z_{ct}}(z_{ct})q_{\beta_r}(\beta_r)q_{\gamma_{r,k}}(\gamma_{r,k})$$

where $\boldsymbol{\theta}$ is the full vector of model parameters, and the optimal approximating partitioned distributions are

$$\begin{aligned} q_{\tilde{y}_{ctr}}^*(\tilde{y}_{ctr}) &\propto \exp[E_{-\tilde{y}_{ctr}} \log p(\tilde{y}_{ctr}|\boldsymbol{\theta}_{-\tilde{y}_{ctr}}, \mathbf{y}_{ctr})] \\ &\sim \mathcal{TN}(E(z_{ct})E(\beta_r), 1, E(\gamma_{r,y_{ctr}-1}), E(\gamma_{r,y_{ctr}})), \\ q_{z_{ct}}^*(z_{ct}) &\propto \exp[E_{-z_{ct}} \log p(z_{ct}|\boldsymbol{\theta}_{-z_{ct}}, \mathbf{y}_{ct})] \\ &\sim \mathcal{N}\left(\frac{\sum_{r \in R_{ct}} E(\beta_r)E(\tilde{y}_{ctr})}{1 + \sum_{r \in R_{ct}} E(\beta_r^2)}, \frac{1}{1 + \sum_{r \in R_{ct}} E(\beta_r^2)}\right), \\ q_{\beta_r}^*(\beta_r) &\propto \exp[E_{-\beta_r} \log p(\beta_r|\boldsymbol{\theta}_{-\beta_r}, \mathbf{y}_r)] \\ &\sim \mathcal{N}\left(\frac{\sum_{c,t \in J_r} E(z_{ct})E(\tilde{y}_{ctr}) + \frac{\mu_\beta}{\sigma_\beta^2}}{\frac{1}{\sigma_\beta^2} + \sum_{c,t \in J_r} E(z_{ct}^2)}, \frac{1}{\frac{1}{\sigma_\beta^2} + \sum_{c,t \in J_r} E(z_{ct}^2)}\right), \\ q_{\gamma_{r,k}}^*(\gamma_{r,k}) &\propto \exp[E_{-\gamma_{r,k}} \log p(\gamma_{r,k}|\boldsymbol{\theta}_{-\gamma_{r,k}}, \mathbf{y}_r)] \\ &\sim U\left(\max\left[-2, \max_{y_{ctr}=k} E(\tilde{y}_{ctr})\right], \min\left[\min_{y_{ctr}=k+1} E(\tilde{y}_{ctr}), 2\right]\right). \end{aligned} \tag{8}$$

The expectations in the approximating distributions are

$$\begin{aligned} E(\tilde{y}_{ctr}) &= \mu_{ctr} + \frac{\phi(E(\gamma_{r,y_{ctr}-1}) - \mu_{ctr}) - \phi(E(\gamma_{r,y_{ctr}}) - \mu_{ctr})}{\Phi(E(\gamma_{r,y_{ctr}}) - \mu_{ctr}) - \Phi(E(\gamma_{r,y_{ctr}-1}) - \mu_{ctr})} \\ E(z_{ct}) &= \frac{\sum_{r \in R_{ct}} E(\beta_r)E(\tilde{y}_{ctr})}{1 + \sum_{r \in R_{ct}} E(\beta_r^2)} \\ E(\beta_r) &= \frac{\sum_{c,t \in J_r} E(z_{ct})E(\tilde{y}_{ctr}) + \frac{\mu_\beta}{\sigma_\beta^2}}{\frac{1}{\sigma_\beta^2} + \sum_{c,t \in J_r} E(z_{ct}^2)} \\ E(\gamma_{r,k}) &= \frac{1}{2} \left(\max\left[-2, \max_{y_{ctr}=k} E(\tilde{y}_{ctr})\right] + \min\left[\min_{y_{ctr}=k+1} E(\tilde{y}_{ctr}), 2\right] \right) \end{aligned}$$

where $\mu_{ctr} = E(z_{ct})E(\beta_r)$. Note that the notation $E(x)$ is shorthand for the expected value of x with respect to the approximating distribution $q_x(x)$. One confusing aspect of these equations is that they appear to be infinitely recursive. So, for example, $E(\tilde{y}_{ctr})$ is a function of $E(z_{ct})$ which is a function of $E(y_{ctr})$. But, at any point in the algorithm, the current estimate of each $q_x(x)$ distribution is fixed and we have an approximation to $E(x)$ for each x that can be plugged directly into whatever distribution function we are updating our estimate at the given step.

The algorithm works by iteratively approximating the values $E(y_{ctr})$, $E(z_{ct})$, $E(\beta_r)$, and $E(\gamma_{r,k})$ across c , t , r , and k . Upon convergence, which we evaluate by monitoring change in the lower bound on the marginal likelihood

$$\underline{p}(\mathbf{y}, q) \equiv \exp \int q(\boldsymbol{\theta}) \log \left[\frac{p(\mathbf{y}, \boldsymbol{\theta})}{q(\boldsymbol{\theta})} \right] d\boldsymbol{\theta},$$

we can calculate approximate point estimates and credible intervals for parameters of interest by plugging our final estimates of these expectations into the approximating conditionals described in equation 8.

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